Contents

1 Physics Vectors **3**
1.1 The Physics Vector Classes . . . 3
1.2 TVector3 . . . 4
1.3 TRotation . . . 6
1.4 TLorentzVector . . . 8
1.5 TLorentzRotation . . . 10

Chapter 1

Physics Vectors

The physics vector classes describe vectors in three and four dimensions and their rotation algorithms. The classes were ported to root from CLHEP see:

http://www.cern.ch/clhep/manual/UserGuide/Vector/vector.html

1.1 The Physics Vector Classes

In order to use the physics vector classes you will have to load the Physics library:

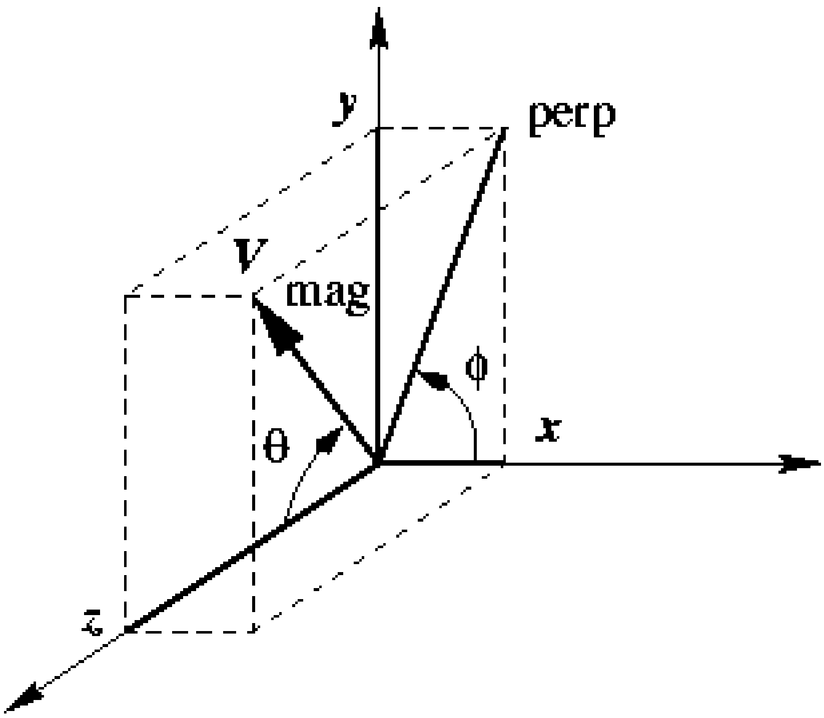
```
gSystem.Load("libPhysics.so");
```

There are four classes in this package. They are:

TVector3 is a general three-vector. A **TVector3** may be expressed in Cartesian, polar, or cylindrical coordinates. Methods include dot and cross products, unit vectors and magnitudes, angles between vectors, and rotations and boosts. There are also functions of particular use to HEP, like pseudo-rapidity, projections, and transverse part of a **TVector3**, and kinetic methods on 4-vectors such as Invariant Mass of pairs or containers of particles.

TLorentzVector is a general four-vector class, which can be used either for the description of position and time (`x`, `y`, `z`, `t`) or momentum and energy (`px`, `py`, `pz`, `E`). **TRotation** is a class describing a rotation of a **TVector3** object. **TLorentzRotation** is a class to describe the Lorentz transformations including Lorentz boosts and rotations. In addition, a **TVector2** is a basic implementation of a vector in two dimensions and is not part of the CLHEP translation.

1.2 TVector3



`TVector3` is a general three-vector class, which can be used for description of different vectors in 3D. Components of three vectors:

- x, y, z = basic components
- θ = azimuth angle
- ϕ = polar angle
- magnitude = $mag = \sqrt{x^2 + y^2 + z^2}$
- transverse component = $perp = \sqrt{x^2 + y^2}$

Using the `TVector3` class, you should remember that it contains only common features of three vectors and lacks methods specific for some particular vector values. For example, it has no translated function because translation has no meaning for vectors.

1.2.1 Declaration / Access to the Components

`TVector3` has been implemented as a vector of three `Double_t` variables, representing the Cartesian coordinates. By default the values are initialized to zero, however you can change them in the constructor:

```
TVector3 v1;        // v1 = (0,0,0)
TVector3 v2(1);     // v2 = (1,0,0)
TVector3 v3(1,2,3); // v3 = (1,2,3)
TVector3 v4(v2);    // v4 = v2
```

It is also possible (but not recommended) to initialize a `TVector3` with a `Double_t` or `Float_t C` array. You can get the components by name or by index:

```
xx = v1.X();    or    xx = v1(0);
yy = v1.Y();          yy = v1(1);
zz = v1.Z();          zz = v1(2);
```

The methods `SetX()`, `SetY()`, `SetZ()` and `SetXYZ()` allow you to set the components:

```
v1.SetX(1.); v1.SetY(2.); v1.SetZ(3.);
v1.SetXYZ(1.,2.,3.);
```

1.2.2 Other Coordinates

To get information on the **TVector3** in spherical (**rho**, **phi**, **theta**) or cylindrical (**z**, **r**, **theta**) coordinates, the following methods can be used.

```
Double_t m  = v.Mag();
// get magnitude (=rho=Sqrt(x*x+y*y+z*z)))
Double_t m2 = v.Mag2();    // get magnitude squared
Double_t t  = v.Theta();   // get polar angle
Double_t ct = v.CosTheta();// get cos of theta
Double_t p  = v.Phi();     // get azimuth angle
Double_t pp = v.Perp();    // get transverse component
Double_t pp2= v.Perp2();   // get transverse squared
```

It is also possible to get the transverse component with respect to another vector:

```
Double_t ppv1  = v.Perp(v1);
Double_t pp2v1 = v.Perp2(v1);
```

The pseudo-rapidity `(eta = -ln (tan (theta/2)))` can be obtained by `Eta()` or `PseudoRapidity()`:

```
Double_t eta = v.PseudoRapidity();
```

These setters change one of the non-Cartesian coordinates:

```
v.SetTheta(.5);  // keeping rho and phi
v.SetPhi(.8);    // keeping rho and theta
v.SetMag(10.);   // keeping theta and phi
v.SetPerp(3.);   // keeping z and phi
```

1.2.3 Arithmetic / Comparison

The **TVector3** class has operators to add, subtract, scale and compare vectors:

```
v3  = -v1;
v1  = v2+v3;
v1 += v3;
v1  = v1 - v3;
v1 -= v3;
v1 *= 10;
v1  = 5*v2;
if(v1 == v2) {...}
if(v1 != v2) {...}
```

1.2.4 Related Vectors

```
v2 = v1.Unit();       // get unit vector parallel to v1
v2 = v1.Orthogonal(); // get vector orthogonal to v1
```

1.2.5 Scalar and Vector Products

```
s = v1.Dot(v2);   // scalar product
s = v1 * v2;      // scalar product
v = v1.Cross(v2); // vector product
```

1.2.6 Angle between Two Vectors

```
Double_t a = v1.Angle(v2);
```

1.2.7 Rotation around Axes

```
v.RotateX(.5);
v.RotateY(TMath::Pi());
v.RotateZ(angle);
```

1.2.8 Rotation around a Vector

```
v1.Rotate(TMath::Pi()/4, v2); // rotation around v2
```

1.2.9 Rotation by TRotation Class

`TVector3` objects can be rotated by `TRotation` objects using the `Transform()` method, the operator `*=`, or the operator `*` of the `TRotation` class. See the later section on `TRotation`.

```
TRotation m;
...
v1.Transform(m);
v1 = m*v1;
v1 *= m;                          // v1 = m*v1
```

1.2.10 Transformation from Rotated Frame

This code transforms v1 from the rotated frame (z' parallel to direction, x' in the theta plane and y' in the `xy` plane as well as perpendicular to the theta plane) to the (x, y, z) frame.

```
TVector3 direction = v.Unit()
v1.RotateUz(direction);       // direction must be TVector3 of unit length
```

1.3 TRotation

The `TRotation` class describes a rotation of `TVector3` object. It is a 3 * 3 matrix of `Double_t`:

$$\begin{vmatrix} xx & xy & xz \\ yx & yy & yz \\ zx & zy & zz \end{vmatrix}$$

It describes a so-called active rotation, i.e. a rotation of objects inside a static system of coordinates. In case you want to rotate the frame and want to know the coordinates of objects in the rotated system, you should apply the inverse rotation to the objects. If you want to transform coordinates from the rotated frame to the original frame you have to apply the direct transformation. A rotation around a specified axis means counterclockwise rotation around the positive direction of the axis.

1.3.1 Declaration, Access, Comparisons

```
TRotation r;        // r initialized as identity
TRotation m(r);     // m = r
```

There is no direct way to set the matrix elements - to ensure that a `TRotation` always describes a real rotation. But you can get the values by with the methods `XX()..ZZ()` or the `(,)` operator:

```
Double_t xx = r.XX();           // the same as xx=r(0,0)
xx = r(0,0);
if (r==m) {...}                 // test for equality
if (r!=m) {..}                  // test for inequality
if (r.IsIdentity()) {...}       // test for identity
```

1.3.2 Rotation around Axes

The following matrices describe counter-clockwise rotations around the coordinate axes and are implemented in: `RotateX()`,`RotateY()` and `RotateZ()`:

$$Rx(a) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & cos(a) & -sin(a) \\ 0 & sin(a) & cos(a) \end{vmatrix} Ry(a) = \begin{vmatrix} cos(a) & 0 & sin(a) \\ 0 & 1 & 0 \\ -sin(a) & 0 & cos(a) \end{vmatrix} Rz(a) = \begin{vmatrix} cos(a) & -sin(a) & 0 \\ sin(a) & cos(a) & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

```
r.RotateX(TMath::Pi()); // rotation around the x-axis
```

1.3.3 Rotation around Arbitrary Axis

The `Rotate()` method allows you to rotate around an arbitrary vector (not necessary a unit one) and returns the result.

```
r.Rotate(TMath::Pi()/3,TVector3(3,4,5));
```

It is possible to find a unit vector and an angle, which describe the same rotation as the current one:

```
Double_t angle;
TVector3 axis;
r.GetAngleAxis(angle,axis);
```

1.3.4 Rotation of Local Axes

The `RotateAxes()method` adds a rotation of local axes to the current rotation and returns the result:

```
TVector3 newX(0,1,0);
TVector3 newY(0,0,1);
TVector3 newZ(1,0,0);
a.RotateAxes(newX,newX,newZ);
```

Methods `ThetaX()`, `ThetaY()`, `ThetaZ()`, `PhiX()`, `PhiY()`, `PhiZ()` return azimuth and polar angles of the rotated axes:

```
   Double_t tx,ty,tz,px,py,pz;
   tx= a.ThetaX();
...
   pz= a.PhiZ();
```

1.3.5 Inverse Rotation

```
   TRotation a,b;
...
   b = a.Inverse();// b is inverse of a, a is unchanged
   b = a.Invert();// invert a and set b = a
```

1.3.6 Compound Rotations

The `operator *` has been implemented in a way that follows the mathematical notation of a product of the two matrices which describe the two consecutive rotations. Therefore, the second rotation should be placed first:

```
r = r2 * r1;
```

1.3.7 Rotation of TVector3

The `TRotation` class provides an `operator *` which allows expressing a rotation of a `TVector3` analog to the mathematical notation:

$$\begin{vmatrix} x' \\ y' \\ z' \end{vmatrix} = \begin{vmatrix} xx & xy & xz \\ yx & yy & yz \\ zx & zy & zz \end{vmatrix} \times \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$

```
TRotation r;
TVector3 v(1,1,1);
v = r * v;
```

You can also use the `Transform()` method or the `operator *=` of the **`TVector3`** class:

```
TVector3 v;
TRotation r;
v.Transform(r);
```

1.4 TLorentzVector

`TLorentzVector` is a general four-vector class, which can be used either for the description of position and time (`x`, `y`, `z`, `t`) or momentum and energy (`px`, `py`, `pz`, `E`).

1.4.1 Declaration

`TLorentzVector` has been implemented as a set a **`TVector3`** and a `Double_t` variable. By default, all components are initialized by zero.

```
TLorentzVector v1;  // initialized by (0.,0.,0.,0.)
TLorentzVector v2(1.,1.,1.,1.);
TLorentzVector v3(v1);
TLorentzVector v4(TVector3(1.,2.,3.),4.);
```

For backward compatibility there are two constructors from a `Double_t` and `Float_t` array.

1.4.2 Access to Components

There are two sets of access functions to the components of a **`TLorentzVector`**: `X()`, `Y()`, `Z()`, `T()` and `Px()`, `Py()`, `Pz()` and `E()`. Both sets return the same values but the first set is more relevant for use where **`TLorentzVector`** describes a combination of position and time and the second set is more relevant where **`TLorentzVector`** describes momentum and energy:

```
   Double_t xx =v.X();
...
   Double_t tt = v.T();
   Double_t px = v.Px();
...
   Double_t ee = v.E();
```

The components of **`TLorentzVector`** can also accessed by index:

```
xx = v(0);orxx = v[0];
yy = v(1);yy = v[1];
zz = v(2);zz = v[2];
tt = v(3);tt = v[3];
```

You can use the `Vect()` method to get the vector component of **`TLorentzVector`**:

```
TVector3 p = v.Vect();
```

For setting components there are two methods: `SetX(),..., SetPx(),...`:

```
   v.SetX(1.); orv.SetPx(1.);
......
   v.SetT(1.);v.SetE(1.);
```

To set more the one component by one call you can use the `SetVect()` function for the **`TVector3`** part or `SetXYZT()`, `SetPxPyPzE()`. For convenience there is also a `SetXYZM()`:

```
v.SetVect(TVector3(1,2,3));
v.SetXYZT(x,y,z,t);
v.SetPxPyPzE(px,py,pz,e);
v.SetXYZM(x,y,z,m);   // v = (x,y,z,e = Sqrt(x*x+y*y+z*z+m*m))
```

1.4.3 Vector Components in Non-Cartesian Coordinates

There are a couple of methods to get and set the **TVector3** part of the parameters in spherical coordinate systems:

```
Double_t m, theta, cost, phi, pp, pp2, ppv2, pp2v2;
m = v.Rho();
t = v.Theta();
cost = v.CosTheta();
phi  = v.Phi();
v.SetRho(10.);
v.SetTheta(TMath::Pi()*.3);
v.SetPhi(TMath::Pi());
```

or get information about the r-coordinate in cylindrical systems:

```
Double_t pp, pp2, ppv2, pp2v2;
pp   = v.Perp();      // get transverse component
pp2  = v.Perp2();     // get transverse component squared
ppv2 = v.Perp(v1);    // get transverse component with respect
                      // to another vector
pp2v2 = v.Perp(v1);
```

there are two more set functions `SetPtEtaPhiE(pt,eta,phi,e)` and `SetPtEtaPhiM(pt,eta,phi,m)` for convenience.

1.4.4 Arithmetic and Comparison Operators

The **TLorentzVector** class provides operators to add subtract or compare four-vectors:

```
v3 = -v1;
v1 = v2+v3;
v1+= v3;
v1 = v2 + v3;
v1-= v3;
if (v1 == v2) {...}
if (v1 != v3) {...}
```

1.4.5 Magnitude/Invariant mass, beta, gamma, scalar product

The scalar product of two four-vectors is calculated with the `(-,-,-,+)`metric:

s = v1*v2 = `t1*t2-x1*x2-y1*y2-z1*z2`

The magnitude squared `mag2` of a four-vector is therefore:

mag2 = v*v = `t*t-x*x-y*y-z*z`

If `mag2` is negative: **mag = -Sqrt(-mag*mag)**. The methods are:

```
Double_t s, s2;
s  = v1.Dot(v2);// scalar product
s  = v1*v2;// scalar product
s2 = v.Mag2();ors2 = v.M2();
s  = v.Mag();s  = v.M();
```

Since in case of momentum and energy the magnitude has the meaning of invariant mass **TLorentzVector** provides the more meaningful aliases `M2()` and `M()`. The methods `Beta()` and `Gamma()` returns `beta` and `gamma = 1/Sqrt(1-beta*beta)`.

1.4.6 Lorentz Boost

A boost in a general direction can be parameterized with three parameters which can be taken as the components of a three vector `b=(bx,by,bz)`. With `x=(x,y,z)` and `gamma=1/Sqrt(1-beta*beta)` (beta being the module of vector b), an arbitrary active Lorentz boost transformation (from the rod frame to the original frame) can be written as:

`x = x' + (gamma-1)/(beta*beta)*(b*x')*b + gamma*t'*b`

`t = gamma(t'+ b*x')`

The `Boost()` method performs a boost transformation from the rod frame to the original frame. `BoostVector()` returns a **`TVector3`** of the spatial components divided by the time component:

```
TVector3 b;
v.Boost(bx,by,bz);
v.Boost(b);
b = v.BoostVector();// b=(x/t,y/t,z/t)
```

1.4.7 Rotations

There are four sets of functions to rotate the **`TVector3`** component of a **`TLorentzVector`**:

Around Axes:

```
v.RotateX(TMath::Pi()/2.);
v.RotateY(.5);
v.RotateZ(.99);
```

Around an arbitrary axis:

```
v.Rotate(TMath::Pi()/4., v1); // rotation around v1
```

Transformation from rotated frame:

```
v.RotateUz(direction); // direction must be a unit TVector3
```

Rotation by **`TRotation`**:

```
TRotation r;
v.Transform(r);//or v *= r;  (v = r*v)
```

1.4.8 Miscellaneous

Angle between two vectors:

```
Double_t a = v1.Angle(v2);// get angle between v1 and v2
```

Methods `Plus()` and `Minus()` return the positive and negative light-cone components:

```
Double_t pcone = v.Plus();
Double_t mcone = v.Minus();
```

A general Lorentz transformation (see class `TLorentzRotation)` can be used by the `Transform()` method, the `*=, or *` operator of the **`TLorentzRotation`** class:

```
TLorentzRotation l;
v.Transform(l);
v = l*v;orv *= l;// v = l*v
```

1.5 TLorentzRotation

The **`TLorentzRotation`** class describes Lorentz transformations including Lorentz boosts and rotations (see **`TRotation`**)

$$lambda = \begin{vmatrix} xx & xy & xz & xt \\ yx & yy & yz & yt \\ zx & zy & zz & zt \\ tx & ty & tz & tt \end{vmatrix}$$

1.5.1 Declaration

By default it is initialized to the identity matrix, but it may also be initialized by other **`TLorentzRotation`**, by a pure **`TRotation`** or by a boost:

```
TLorentzRotation l;   // l is initialized as identity
TLorentzRotation m(l);// m = l
TRotation r;
TLorentzRotation lr(r);
TLorentzRotation lb1(bx,by,bz);
TVector3 b;
TLorentzRotation lb2(b);
```

The Matrix for a Lorentz boosts is:

$$\begin{vmatrix} 1+gamma'*bx*bx & gamma'*bx*by & gamma'*bx*bz & gamma*bx \\ gamma'*bx*bz & 1+gamma'*by*by & gamma'*by*by & gamma*by \\ gamma'*bz*bx & gamma'*bz*by & 1+gamma'*bz*bz & gamma*bz \\ gamma*bx & gamma*by & gamma*bz & gamma \end{vmatrix}$$

with the boost vector `b=(bx,by,bz)`; `gamma=1/Sqrt(1-beta*beta)`;`gamma'=(gamma-1)/beta*beta.`

1.5.2 Access to the Matrix Components/Comparisons

The access to the matrix components is possible with the methods `XX()`, `XY()` ... `TT()`, and with the operator`(int,int)`:

```
Double_t xx;
TLorentzRotation l;
xx = l.XX();              // gets the xx component
xx = l(0,0);              // gets the xx component
if (l == m) {...}         // test for equality
if (l != m) {...}         // test for inequality
if (l.IsIdentity()) {...} // test for identity
```

1.5.3 Transformations of a Lorentz Rotation

There are four possibilities to find the product of two **`TLorentzRotation`** transformations:

```
TLorentzRotation a,b,c;
c = b*a;                       // product
c = a.MatrixMultiplication(b); // a is unchanged
a *= b;                        // a=a*b
c = a.Transform(b)             // a=b*a then c=a
```

Lorentz boosts:

```
Double_t bx, by, bz;
TVector3 v(bx,by,bz);
TLorentzRotation l;
l.Boost(v);
l.Boost(bx,by,bz);
```

Rotations:

```
TVector3 axis;
l.RotateX(TMath::Pi()); // rotation around x-axis
l.Rotate(.5,axis);      // rotation around specified vector
```

Inverse transformation: use the method `Inverse()`to return the inverse transformation keeping the current one unchanged. The method `Invert()` inverts the current **`TLorentzRotation`**:

```
l1 = l2.Inverse(); // l1 is inverse of l2, l2 unchanged
l1 = l2.Invert();  // invert l2, then  l1=l2
```

The matrix for the inverse transformation of a **`TLorentzRotation`** is as follows:

$$\begin{vmatrix} xx & xy & xz & -tx \\ yx & yy & yz & -ty \\ zx & zy & zz & -tz \\ -xt & -yt & -zt & tt \end{vmatrix}$$

1.5.4 Transformation of a TLorentzVector

To apply **TLorentzRotation** to **TLorentzVector** you can use either the `VectorMultiplication()` method or the `* operator`. You can also use the `Transform()` function and the `*=`operator of the class **TLorentzVector**.

```
   TLorentzVector v;
   TLorentzRotation l;
...
   v = l.VectorMultiplication(v);
   v = l * v;
   v.Transform(l);
   v *= l;            // v = l*v
```

1.5.5 Physics Vector Example

The test file `$ROOTSYS/test/TestVectors.cxx is` an example of using physics vectors. The vector classes are not loaded by default, and to run it, you will need to load `libPhysics.so` first:

```
root[] .L $ROOTSYS/lib/libPhysics.so
root[] .x TestVectors.cxx
```

To load the physics vector library in a ROOT application use:

```
   gSystem->Load("libPhysics");
```

The example `$ROOTSYS/test/TestVectors.cxx` does not return much, especially if all went well, but when you look at the code you will find examples for many calls.