# Proposal for a C++ MATHLIB

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## 1 Introduction

Following the discussions at the Architects Forum in October and our meeting of November 14, this paper is an attempt to come with a more concrete proposal for the implementation of a Math Library in C++. This proposal does not exclude in any way the support for existing libraries such as CLHEP [3], GSL [2] or CERNLIB [1]. Whatever decision is taken, these libraries will have to be maintained (at least for CLHEP) for some time in the future to guarantee a smooth transition to the new system. Note that the title says "Math Library in C++" and not in C. A C++ library that would be just a simple wrapper to C functions is not appropriate in most cases. The wrapper does not introduce new functionality, but it introduces a performance overhead. The main reasons to have a true C++ library are:

1. We want to interact with real objects (data and algorithms), not just algorithms.
2. We want to provide higher level interfaces hidding the implementation details (algorithms). A true Object-Oriented API should remain stable if internal storage or algorithms change. One can imagine the Mathlib classes being improved over time, or adapted to standard algorithms that could come with the new C++ versions.
3. Many classes require a good graphics interface. A large subset of CERNLIB or GSL has to do with functions. Visualising a function requires to know some of its properties, eg singularities or asymptotic behaviours. This does not mean that the function classes must have built-in graphics. But they must be able to call graphics service classes able to exploit the algorithms in the functions library.
4. Many objects need operators (matrices, vectors, physics vectors, etc).
5. We want to embed these objects in a data model. Users start to request that the math library takes care of memory management and/or persistence of the object . See for instance the LHC-feedback [5], where persistence of the CLHEP was requested. The user would like to save and restore random-generator seeds etc .
6. We want to have an interactive interface from our interpreters, hence a dictionary.

GSL is a rich library, but with a user interface very similar to a Fortran library. The data have to be specified via arrays or special C-structs in all the algorithms. The GSL functions do not carry an internal state. They are at a very low level. GSL++ has been an attempt to implement a true C++ interface to GSL. GSL++ is stalled because of a limited interest if points 3, 4, 5 and 6 above are not implemented. CLHEP has been an attempt to implement a mini true C++ library fulfilling also conditions 1 and 2, but not the essential conditions 3 and 4. The current CLHEP is also a very small subset of what is required.

**Object-Oriented API vs Procedural API**

```
gsl style  : double gsl_sf_gamma(double x)
             int gsl_sf_gamma_e(double x, gsl_sf_result* result)

root style : TF1 gamma("gamma",TMath::Gamma,0,1)
             gamma.Draw()
             gamma.Eval(x)
             gamma.Derivative(x)
             gamma.Integral(0.1,0.3)
```

**Proposal**

The library could be structured in 3 logical groups:

A : a set of often used algorithms (like TMath) grouped into a few classes with static functions only. The corresponding shared lib libMathlib will be required by all applications and could in turn be embedded into the core libraries (like the ROOT libCore) to minimize the number of libraries that a user has to specify on the link command line.

B : a set of classes for frequently used objects like the CLHEP Physics vectors classes, random generators or matrices (or the ROOT equivalent), the parametric functions, the minimization classes. Each set of these classes will have its own shared library, eg libRandom, libMatrix, libPhysics, libMinuit, libGMinuit, libFumili, etc. These libraries are typically dynamically linked by the plugin manager.

C : a set of algorithms, like in case A, but less frequently used. Again, mostly static functions in a restricted number of utility classes. These classes could be grouped into a large library libMathlib2.

Each shared library above must include the class dictionaries to minimize dependencies on parallel libraries or possible mismatches. The dictionary is anyway essential for all the shared libraries in the group B. The CVS structure should reflect the proposed structure with a directory for A, a directory for each main component of B and one or a few directories for C. Following a detailed analysis of the tables in the following sections, we estimate that

a large subset (more than 60 per cent) is already in the ROOT libraries. We estimate the work to implement A and B to be of the order of a few weeks. Eddy Offermann is currently polishing an upgraded Matrix package including a nice test suite. Implementation of C will take more time and could be gradually implemented in the coming year. We are therefore proposing to host the Mathlib structure as a component of the ROOT project. This will have immediate advantages, large user base, infrastructure, ready solution for I/O and interactivity.

This proposal suggests the implementation of a true C++ library fulfilling all the conditions above and reusing existing code or algorithms. Reusing existing code may be achieved by importing and adapting C code in GSL, C++ code from CLHEP, or translating Fortran code from CERNLIB. Like the old CERNLIB, the mathematical library should be a compact set of functions that will run on all supported platforms. It should provide the mathematical backbone for all phases of data analysis from large batch jobs to interactive sessions on a personal computer . The advantages of consistent use of the same mathematical library throughout the analysis are obvious . However, the choice of the functions inside this package are less obvious. At a first glance, it would seem preferable to start from the GSL code. However, the current CERNLIB does reflect the *Reader's Digest* of mathematical functionality in High-Energy Physics. For many years CERN staff and users have added the necessary functionality.

Obviously, not all functionality is available in the CERNLIB . But future LHC and past LEP analysis-methodology are not that far apart, that it is safe to say that it did and will cover 95% of the necessary functionality. Therefore, we decided to take the current CERNLIB as a benchmark for completeness. When considering whether to endorse a package from *outside*, we should carefully analyze its contents. It is not desirable to end up in a situation where the ratio between *useful* routines and *missing* ones becomes unfavorable . In the next few sections we list the functionality of the CERNLIB, GSL, CLHEP and ROOT [4] . In the tables there will be several columns, labeled *Available* . It is supposed to indicate whether the functionality of the library under consideration is available in the other contenders. The symbols in these columns have the following meaning:

C : CERNLIB

G : GSL

H : CLHEP

R : ROOT basic Root (mainly TMath)

R* : ROOT as a framework

As indicated above, two types of entry ($R$ and $R*$) exist for the ROOT library. The functions tagged with $R*$ have been embedded in the ROOT framework, thereby providing persistence, graphing etc.

## 2 cernlib

In the table below the *availability* column has for CERN (which has by definition an entry for every routine) sometimes an entry $c$ instead of the $C$ . This indicates that its priority is low and could be taken on request.

| B - Elementary Functions | | | | | | |
|---|---|---|---|---|---|---|
| routine | Available | | | | description | # |
| PRMFCT (B002) | c | | | | Prime Numbers Factor Decomposition | 222 |
| RBINOM (B100) | C | | | | Binomial Coefficient | 19 |
| RPLNML (B105) | C | R* | G | | Value of a Polynomial | 50 |

| C - Equations and Special Functions | | | | | | |
|---|---|---|---|---|---|---|
| routine | Available | | | | description | # |
| RSNLEQ (C201) | G | | G | | Systems of Nonlinear Equations | 248 |
| RMULLZ (C202) | C | | G | | Zeros of a Real Polynomial | 159 |
| RZERO (C205) | R* | R* | G | | Zero of a Function of One Real Variable | 138 |
| RRTEQ3 (C207) | C | | G | | Roots of a Cubic Equation | 110 |
| RRTEQ4 (C208) | C | | | | Roots of a Quartic Equation | 137 |
| CPOLYZ (C209) | c | | | | Zeros of a Complex Polynomial | 264 |
| NZERFZ (C210) | c | | | | Number of Zeros of a Complex Function | 93 |
| ERF (C300) | R | R | G | | Error Function and Complementary Error | 80 |
| FREQ (C301) | R | R | G | | Normal Frequency Function | 52 |
| GAMMA (C302) | R | R | G | | Gamma Function for Positive Argument | 81 |
| GAMMF (C303) | R | R | G | | Gamma Function for Real Argument | 81 |
| ALGAMA (C304) | R | R | G | | Logarithm of the Gamma Function | 122 |
| CGAMMA (C305) | c | | G | | Gamma Function for Complex Argument | 98 |
| CLGAMA (C306) | R | R | G | | Logarithm of the Gamma Function | 31 |
| CCLBES (C309) | c | | G | | Coulomb Wave, Bessel, and Sphe Bessel | 820 |
| BESJ0 (C312) | R | R | G | | Bessel Funct J and Y Orders 0 and 1 | 331 |
| BESI0 (C313) | R | R | G | | Modified Bessel Funct I and K 0 and 1 | 351 |
| RRIZET (C315) | c | | G | | Riemann Zeta Function | 164 |
| RPSIPG (C316) | c | | G | | Psi (Digamma) and Polygamma Functions | 312 |
| CPSIPG (C317) | c | | G | | Psi (Digamma) and Polygamma Functions | 20 |
| RELFUN (C318) | c | | G | | Jacobian Elliptic Functions real | 23 |
| CELFUN (C320) | c | | G | | Jacobian Elliptic Functions complex | 126 |
| CGPLG (C321) | c | | | | Nielsen's Generalized Polylogarithm | 410 |

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| routine | Available | | | | description | # |
|---|---|---|---|---|---|---|
| RFRSIN (C322) | c | | | | Fresnel Integrals | 196 |
| RFERDR (C323) | c | | G | | Fermi-Dirac Function | 148 |
| DATANI (C324) | c | | G | | Arctangent integral | 19 |
| RCLAUS (C326) | c | | G | | Clausen Function | 86 |
| BSIR4 (C327) | c | | G | | Modified Bessel F I and K 1/4,1/2,3/4 | 360 |
| CWHITM (C328) | c | | | | Whittaker Funct M of Complex Argument | 125 |
| RASLGF (C330) | c | | G | | Legendre,Associated Legendre Functions | 25 |
| RFCONC (C331) | c | | G | | Conical Functions of the First Kind | 26 |
| RDILOG (C332) | R | R | G | | Dilogarithm Function | 25 |
| RGAPNC (C334) | c | | G | | Incomplete Gamma Functions | 33 |
| CWERF (C335) | c | | G | | Complex Error Function | 74 |
| RSININ (C336) | c | | | | Sine and Cosine Integrals | 196 |
| REXPIN (C337) | c | | G | | Exponential Integral | 27 |
| CEXPIN (C338) | c | | G | | Exponential Integral Complex Argument | 27 |
| RDAWSN (C339) | C | | G | | Dawson's Integral | 96 |
| BSIR3 (C340) | c | | G | | Modified Bessel Functions I,K 1/3, 2/3 | 343 |
| BSKA (C341) | R | R | G | | Modified Bessel Functions K | 130 |
| RSTRH0 (C342) | R | R | G | | Struve Funct of Orders 0 and 1 | 195 |
| BSJA (C343) | R | R | G | | Bessel Funct J and I with Pos Arg | 181 |
| CBSJA (C344) | c | | G | | Bessel Funct J with Complex Argument | 60 |
| RBZEJY (C345) | c | | G | | Zeros of Bessel Functions J and Y | 223 |
| RELI1 (C346) | c | | G | | Elliptic Integrals of 1st, 2nd, 3rd | 67 |
| RELI1C (C347) | c | | G | | Elliptic Integrals 1st, 2nd, 3rd | 140 |
| CELINT (C348) | c | | G | | Elliptic Integral for Complex Argument | 151 |
| RTHETA (C349) | c | | | | Jacobian Theta Functions | 289 |

## D - Integration, Minimization, Non-linear Fitting

| routine | Available | | | | description | # |
|---|---|---|---|---|---|---|
| SIMPS (D101) | C | | | | Integration by Simpson's Rule | 20 |
| RADAPT (D102) | R* | R* | G | | Adaptive Gaussian Quadrature | 95 |
| GAUSS (D103) | R* | R* | G | | Adaptive Gaussian Quadrature | 18 |
| RCAUCH (D104) | c | | G | | Cauchy Principal Value Integration | 24 |
| RTRINT (D105) | c | | | | Integration over a Triangle | 24 |
| RGS56P (D106) | R* | R* | | | Gauss Quadrature with 5 and 6 Points | 42 |
| RGQUAD (D107) | R* | R* | | | N-Point Gaussian Quadrature | 18 |
| TRAPER (D108) | C | | | | Trapezoidal Rule Integration | 56 |
| RGMLT (D110) | C | R* | G | | Gauss Quad for Multiple Integrals | 26 |

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| routine | Available | | | | description | # |
|---|---|---|---|---|---|---|
| CGAUSS (D113) | c | | | | Complex Integ Along a Line Segment | 18 |
| RIWIAD (D114) | c | | | | Multidim Monte-Carlo Integration | 374 |
| RADMUL (D120) | c | | G | | Quadrature for Multiple Integrals | 224 |
| DIVON4 (D151) | R* | R* | | | Multidim Integration/Random Numbers | ? |
| RRKSTP (D200) | c | | G | | 1st-ord D Eq (Runge–Kutta) | 12 |
| RDEQBS (D201) | c | | G | | 1st-ord D Eq (Gragg–Bulirsch–Stoer) | 22 |
| RDEQMR (D202) | c | | G | | 1st-ord D Eq (Runge–Kutta–Merson) | 23 |
| RRKNYS (D203) | c | | G | | 2nd-ord D Eq (Runge–Kutta–Nystrm) | 12 |
| EPDE1 (D300) | c | | G | | Elliptic Partial D Equation | 134 |
| ELPAHY (D302) | c | | G | | Fast Partial D Eq Solver | 125 |
| RDERIV (D401) | c | | G | | Numerical Differentiation | 19 |
| LEAMAX (D501) | c | | | | Non-Lin Least Sq & Max Likelihood | 2084 |
| RMINFC (D503) | R* | | G | | Mini of a Function of One Variable | 73 |
| RFRDH1 (D601) | c | | | | Lin Fredholm Integ Eq of 2nd Kind | 25 |
| RFT (D700) | c | | G | | Real Fast Fourier Transform | 551 |
| CFT (D702) | c | | G | | Complex Fast Fourier Transform | 576 |
| CFSTFT (D706) | c | | G | | Complex Fast Fourier Transform | 67 |

# E- Interpolation, Approximations, Linear Fitting

| routine | Available | | | | description | # |
|---|---|---|---|---|---|---|
| POLINT (E100) | C | | G | | Polynomial Interpolation | ? |
| MAXIZE (E102) | R | R | G | | Maximum/Minimum Elements of Arrays | 61 |
| AMAXMU (E103) | R | R | G | | Largest Number in Scattered Vector | ? |
| FINT (E104) | c | | G | | Multidim Linear Interpolation | ? |
| DIVDIF (E105) | C | | G | | Function Interpolation | ? |
| LOCATR (E106) | R | R | G | | Binary Search in Ord Array | ? |
| RLSQPM (E201) | R* | R* | G | | Least Squares Polynomial Fit | 76 |
| LSQ (E208) | R* | R* | G | | Least Squares Polynomial Fit | ? |
| NORBAS (E210) | C | | G | | Polynom Splines / Norm B-Splines | 3417 |
| RCSPLN (E211) | C | R* | G | | Cubic Splines and their Integrals | 20 |
| LFIT (E250) | R* | R* | G | | Least-Squar Fit to Straight Line | 60 |
| PARLSQ (E255) | R* | R* | G | | Least-Squar Fit to Parabola | 132 |
| RCHECF (E406) | c | | G | | Chebyshev Coeff of a Function | 40 |
| RCHSUM (E407) | c | | G | | Summation of Chebyshev Series | ? |
| RTRGSM (E409) | c | | G | | Summation of Trig Series | 42 |

| F- Matrices, Vectors and Linear Equations | | | | | | |
|---|---|---|---|---|---|---|
| routine | Available | | | | description | # |
| LAPACK (F001) | R* | R* | G | H | Linear Algebra Package | |
| RVADD (F002) | R* | R* | G | H | Elementary Vector Processing | |
| RMADD (F003) | R* | R* | G | H | Elementary Matrix Processing | |
| RMMLT (F004) | R* | R* | G | H | Matrix Multiplication | |
| RINV (F010) | R* | R* | G | H | Linear Equations, Matrix Inversion | |
| RSINV (F012) | R* | R* | G | H | Symmetric Pos-Def Linear Systems | |
| POLROT (F105) | c | | | | Rotate a 3-Dim Polar Coord System | |
| MXPACK (F110) | R* | R* | | H | TC Matrix Manipulation Package | |
| TR (F112) | R* | R* | G | H | Triangular and Symmetric Matrices | |
| DOTI (F116) | R* | R* | G | H | Scalar Prod of 2 Space-Time Vectors | |
| CROSS (F117) | R* | R* | G | H | Vector Prod of 2 3-Vectors | |
| ROT (F118) | R* | R* | | H | Rotating a 3-Vector | |
| VECMAN (F121) | R* | R* | G | H | Vector Algebra | |
| BVSL (F123) | R* | R* | G | | Bit Vector Manipulation Package | |
| MXDIPR (F150) | c | | | | Direct or Tensor Matrix Product | |
| RBEQN (F406) | c | | | | Banded Linear Equations | |
| RLHOIN (F500) | c | | | | Linear Homogeneous Inequalities | |

| G - Statistical Analysis and Probability | | | | | | |
|---|---|---|---|---|---|---|
| routine | Available | | | | description | # |
| PROB (G100) | R | R | G | H | Upper Tail Prob of Chi-Squared Dist | 108 |
| CHISIN (G101) | R | R | G | H | Inverse of Chi-Square Distribution | 81 |
| PROBKL (G102) | R | R | | | Kolmogorov Distribution | 51 |
| TKOLMO (G103) | R | R | | | Kolmogorov Test | 66 |
| STUDIS (G104) | C | | | | Student Dist and Its Inverse | 46 |
| GAUSIN (G105) | C | | | | Inverse of Normal Freq Function | 41 |
| GAMDIS (G106) | R | R | G | H | Gamma Distribution | 128 |
| LANDAU (G110) | R | R | G | H | Landau Distribution | 92 |
| VAVLOV (G115) | c | | | | Vavilov Dist and its Inverse | 920 |
| VVILOV (G116) | c | | | | Vavilov Density & Dist Functions | 150 |

| M - Data Handling | | | | | | |
|---|---|---|---|---|---|---|
| routine | Available | | | | description | # |
| SORTZV (M101) | R | R | G | | Sort One-Dim Array | |
| FLPSOR (M103) | R | R | G | | Sort One-Dim Array into Itself | |
| SORCHA (M104) | R | R | G | | Sort One-Dim Character Array | |
| SORTR (M107) | R* | R* | G | | Sort Rows of a Matrix | |
| SORTRQ (M109) | R* | R* | G | | Sort Rows of a Matrix | |
| LOCBYT (M428) | R* | R* | | | Search for Byte-Content | |
| NUMBIT (M429) | R* | R* | | | Number of One-Bits in a Word | |
| GETBIT (M437) | R* | R* | | | Set or Retrieve a Bit in a String | |
| BTMOVE (M438) | R* | R* | | | Move Bit String | |
| GETBYT (M439) | R* | R* | | | Set or Retrieve a Bit String | |
| BITPAK (M441) | R* | R* | | | Handling Bits and Bytes | |
| UBITS (M503) | R* | R* | | | Locate the One-Bits of a Word | |

| U - Quantum Mechanics, Particle Physics | | | | | | |
|---|---|---|---|---|---|---|
| routine | Available | | | | description | # |
| LOREN4 (U101) | R* | R* | | H | Lorentz Transformation | |
| LORENF (U102) | R* | R* | | H | Lorentz Transformations | |
| RWIG3J (U111) | c | | G | | Wigner 3-j, 6-j, 9-j Symbols | |
| RTCLGN (U112) | c | | | | Clebsch-Gordan Coeff | |
| RDJMNB (U501) | c | | | | Beta-Term in Wigner's D-Function | |

| V - Random Numbers and General Purpose Utilities | | | | | | |
|---|---|---|---|---|---|---|
| routine | Available | | | | description | # |
| RNDM (V104) | R* | R* | G | H | Uniform Random Numbers | |
| NRAN (V105) | R* | R* | G | H | Arrays of Uniform Random Numbers | |
| RANMAR (V113) | c | | G | H | Fast Uniform Random Number Generator | |
| RANECU (V114) | R* | R* | G | H | Uniform Random Number Generator | |
| RANLUX (V115) | R* | R* | G | H | Uniform Random Numbers of Quality | |
| RNORML (V120) | R* | R* | G | H | Gauss-dist Random Numbers | |
| CORSET (V122) | R* | R* | G | H | Corr Gauss-dist Random Numbers | |
| RAN3D (V130) | c | | | | Random 3-Dimensional Vectors | |
| RN3DIM (V131) | c | | | | Random 2 and 3-Dimensional Vectors | |

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| routine | Available | | | | description | # |
|---|---|---|---|---|---|---|
| RNGAMA (V135) | c | | G | | Gamma or Chi-Square Random Numbers | |
| RNPSSN (V136) | R* | R* | G | H | Poisson Random Numbers | |
| RNBNML (V137) | R* | R* | G | H | Binomial Random Numbers | |
| RNMNML (V138) | C | | G | | Multinomial Random Numbers | |
| RNHRAN (V149) | R* | R* | G | | Rand Numbers Acc to Any Histogram | |
| HISRAN (V150) | R* | R* | G | | Rand Numbers Acc to Any Histogram | |
| FUNRAN (V151) | R* | R* | G | | Rand Numbers Acc to Any Function | |
| FUNLUX (V152) | R* | R* | G | | Rand Numbers Acc to Any Function | |
| PERMU (V202) | c | | G | | Permutations and Combinations | |
| PROXIM (V306) | c | | | | Adjusting an Angle to Another Angle | |
| GRAPH (V401) | c | | | | Find Compatible Node-Nets in Graph | |

## W - High Energy Physics Simulation, Kinematics, Phase Space

| routine | Available | | | | description | # |
|---|---|---|---|---|---|---|
| GENBOD (W515) | R* | R* | | | N-Body Monte-Carlo Event Generator | |

## Z - Miscellaneous System-Dependent Facilities

| routine | Available | | | | description | # |
|---|---|---|---|---|---|---|
| DATIME (Z007) | R* | R* | | | Job Time and Date | |
| CALDAT (Z009) | R* | R* | | | Calendar Date Conversion | |

# 3 gsl

In the GSL table a new column *action* is introduced . The symbols in this column have the following meaning:

> we need this functionality (but already provided elsewhere)

\+ we could take it from gsl source

## Mathematical Functions

| action | description | Available | | |
|---|---|---|---|---|
| > | Mathematical Constants | R | | |
| > | Infinities and Not-a-number | R | | |
| > | Elementary Functions | R | | |
| | Small integer powers | | | |
| | Testing the Sign of Numbers | | | |
| | Testing for Odd and Even Numbers | | | |
| > | Maximum and Minimum functions | R | | |
| | Approximate Comparison of Floating Point Numbers | | | |

## Complex Numbers

| | | | | |
|---|---|---|---|---|
| + | Complex arithmetic operators | | | |
| + | Elementary Complex Functions | | | |
| + | Complex Trigonometric Functions | | | |
| + | Inverse Complex Trigonometric Functions | | | |
| + | Complex Hyperbolic Functions | | | |
| + | Inverse Complex Hyperbolic Functions | | | |

## Polynomials

| | | | | |
|---|---|---|---|---|
| > | Polynomial Evaluation | C | | |
| | Divided Difference Representation of Polynomials | | | |
| > | Quadratic Equations | C | | |
| > | Cubic Equations | C | | |
| + | General Polynomial Equations | | | |

## Special Functions

| | | | | |
|---|---|---|---|---|
| | Usage | | | |
| | The gsl_sf_result struct | | | |
| | Modes | | | |
| | Airy Functions and Derivatives | | | |

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| action | description | Available | | |
|---|---|---|---|---|
| | Airy Functions | | | |
| | Derivatives of Airy Functions | | | |
| | Zeros of Airy Functions | | | |
| | Zeros of Derivatives of Airy Functions | | | |
| | **Bessel Functions** | | | |
| | Regular Cylindrical Bessel Functions | | | |
| | Irregular Cylindrical Bessel Functions | | | |
| | Regular Modified Cylindrical Bessel Functions | | | |
| | Irregular Modified Cylindrical Bessel Functions | | | |
| > | Regular Spherical Bessel Functions | R | C | |
| > | Irregular Spherical Bessel Functions | R | C | |
| > | Regular Modified Spherical Bessel Functions | R | C | |
| > | Irregular Modified Spherical Bessel Functions | | C | |
| > | Regular Bessel Function - Fractional Order | | C | |
| > | Irregular Bessel Functions - Fractional Order | | C | |
| > | Regular Modified Bessel Functions - Fractional Order | | C | |
| > | Irregular Modified Bessel Functions - Fractional Order | C | | |
| > | Zeros of Regular Bessel Functions | | | |
| | Clausen Functions | | | |
| | **Coulomb Functions** | | | |
| | Normalized Hydrogenic Bound States | | | |
| | Coulomb Wave Functions | | C | |
| | Coulomb Wave Function Normalization Constant | | | |
| | **Coupling Coefficients** | | | |
| | 3-j Symbols | | C | |
| | 6-j Symbols | | C | |
| | 9-j Symbols | | C | |
| | Dawson Function | | | |
| | Debye Functions | | | |
| | **Dilogarithm** | | | |
| > | Real Argument | R | | |
| > | Complex Argument | | C | |
| | Elementary Operations | | | |
| | **Elliptic Integrals** | | | |
| > | Definition of Legendre Forms | | C | |
| | Definition of Carlson Forms | | C | |
| > | Legendre Form of Complete Elliptic Integrals | | C | |

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| action | description | Available | | |
|---|---|---|---|---|
| + | Legendre Form of Incomplete Elliptic Integrals | | | |
| | Carlson Forms | | C | |
| | Elliptic Functions (Jacobi) | | C | |
| | **Error Functions** | | | |
| > | Error Function | R | C | |
| > | Complementary Error Function | R | C | |
| > | Log Complementary Error Function | R | | |
| > | Probability functions | R | C | |
| | **Exponential Functions** | | | |
| > | Exponential Function | R | | |
| > | Relative Exponential Functions | R | | |
| + | Exponentiation With Error Estimate | | | |
| | **Exponential Integrals** | | | |
| > | Exponential Integral | R* | C | |
| | Ei(x) | | | |
| > | Hyperbolic Integrals | R* | C | |
| | Ei_3(x) | | | |
| > | Trigonometric Integrals | R* | C | |
| > | Arctangent Integral | | | |
| | **Fermi-Dirac Function** | | | |
| > | Complete Fermi-Dirac Integrals | | C | |
| > | Incomplete Fermi-Dirac Integrals | | C | |
| > | Gamma Function | | | |
| | Gegenbauer Functions | | | |
| | Hypergeometric Functions | | | |
| > | Laguerre Functions | | C | |
| | Lambert W Functions | | | |
| | **Legendre Functions and Spherical Harmonics** | | | |
| > | Legendre Polynomials | | C | |
| > | Associated Legendre Polynomials and Spherical Harmonics | | C | |
| > | Conical Functions | | C | |
| | Radial Functions for Hyperbolic Space | | | |
| > | Logarithm and Related Functions | | | |
| | Power Function | | | |
| | **Psi (Digamma) Function** | | | |
| > | Digamma Function | R | C | |

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| action | description | Available | | |
|---|---|---|---|---|
| | Trigamma Function | | | |
| | Polygamma Function | | | |
| | Synchrotron Functions | | | |
| | Transport Functions | | | |
| | **Trigonometric Functions** | | | |
| | Circular Trigonometric Functions | | | |
| | Trigonometric Functions for Complex Arguments | | | |
| | Hyperbolic Trigonometric Functions | | | |
| | Conversion Functions | | | |
| | Restriction Functions | | | |
| | Trigonometric Functions With Error Estimates | | | |
| | **Zeta Functions** | | | |
| | Riemann Zeta Function | | C | |
| | Hurwitz Zeta Function | | | |
| | Eta Function | | | |
| **Vectors and Matrices** | | | | |
| | Data types | | | |
| | **Blocks** | | | |
| | Block allocation | | | |
| | Reading and writing blocks | | | |
| | **Vectors** | | | |
| > | Vector allocation | R* | | |
| > | Accessing vector elements | R* | | |
| > | Initializing vector elements | R* | | |
| > | Reading and writing vectors | R* | | |
| > | Vector views | R* | | |
| > | Copying vectors | R* | | |
| > | Exchanging elements | R* | | |
| > | Vector operations | R* | | |
| > | Finding maximum and minimum elements of vectors | R* | | |
| | Vector properties | | | |
| | **Matrices** | | | |
| > | Matrix allocation | R* | | |
| > | Accessing matrix elements | R* | | |
| | Initializing matrix elements | | | |
| > | Reading and writing matrices | R* | | |

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| action | description | Available | | |
|---|---|---|---|---|
| > | Matrix views | R* | | |
| > | Creating row and column views | R* | | |
| > | Copying matrices | R* | | |
| > | Copying rows and columns | R* | | |
| > | Exchanging rows and columns | R* | | |
| > | Matrix operations | R* | | |
| > | Finding maximum and minimum elements of matrices | | | |
| | Matrix properties | | | |

## Permutations

| | | | | |
|---|---|---|---|---|
| | The Permutation struct | | | |
| | Permutation allocation | | | |
| | Accessing permutation elements | | | |
| | Permutation properties | | | |
| | Permutation functions | | | |
| | Applying Permutations | | | |
| | Reading and writing permutations | | | |
| | Permutations in Cyclic Form | | | |

## Combinations

| | | | | |
|---|---|---|---|---|
| | The Combination struct | | | |
| | Combination allocation | | | |
| | Accessing combination elements | | | |
| | Combination properties | | | |
| | Combination functions | | | |
| | Reading and writing combinations | | | |

## Sorting

| | | | | |
|---|---|---|---|---|
| > | Sorting objects | R* | | |
| > | Sorting vectors | R* | C | |
| > | Selecting the k smallest or largest elements | R* | C | |
| > | Computing the rank | R* | | |

## BLAS Support

| | | | | |
|---|---|---|---|---|
| | GSL BLAS Interface | | | |
| | Level 1 | | | |
| | Level 2 | | | |
| | Level 3 | | | |

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| action | description | Available | | |
|---|---|---|---|---|
| **Linear Algebra** | | | | |
| > | LU Decomposition | R* | C | |
| > | QR Decomposition | R* | C | |
| > | QR Decomposition with Column Pivoting | R* | C | |
| > | Singular Value Decomposition | R* | C | |
| > | Cholesky Decomposition | R* | C | |
| > | Tridiagonal Decomposition of Real Symmetric Matrices | R* | C | |
| > | Tridiagonal Decomposition of Hermitian Matrices | R* | C | |
| > | Bidiagonalization | R* | C | |
| > | Householder Transformations | R* | C | |
| | Householder solver for linear systems | | C | |
| | Tridiagonal Systems | | C | |
| **Eigensystems** | | | | |
| > | Real Symmetric Matrices | R* | C | |
| | Complex Hermitian Matrices | | C | |
| > | Sorting Eigenvalues and Eigenvectors | R* | C | |
| **Fast Fourier Transforms (FFTs)** | | | | |
| > | Radix-2 FFT routines for complex data | | C | |
| + | Mixed-radix FFT routines for complex data | | | |
| | Overview of real data FFTs | | | |
| > | Radix-2 FFT routines for real data | | C | |
| + | Mixed-radix FFT routines for real data | | | |
| **Numerical Integration** | | | | |
| > | QNG non-adaptive Gauss-Kronrod integration | R* | C | |
| > | QAG adaptive integration | R* | C | |
| > | QAGS adaptive integration with singularities | R* | C | |
| + | QAGP adaptive integration with known singular points | | | |
| + | QAGI adaptive integration on infinite intervals | | | |
| > | QAWC adaptive integration for Cauchy principal values | R* | C | |
| + | QAWS adaptive integration for singular functions | | | |
| + | QAWO adaptive integration for oscillatory functions | | | |
| + | QAWF adaptive integration for Fourier integrals | | | |
| **Random Number Generation** | | | | |
| > | Random number generator initialization | R* | C | H |
| > | Sampling from a random number generator | R* | C | |

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| action | description | Available | | |
|---|---|---|---|---|
| > | Auxiliary random number generator functions | R* | C | H |
| | Random number environment variables | | | |
| | Copying random number generator state | | | |
| > | Reading and writing random number generator state | R* | | |
| | Random number generator algorithms | | | |
| | Unix random number generators | | | |
| | Other random number generators | | | |

## Quasi-Random Sequences

| | | | | |
|---|---|---|---|---|
| | Quasi-random number generator initialization | | | |
| | Sampling from a quasi-random number generator | | | |
| | Auxiliary quasi-random number generator functions | | | |
| | Saving and resorting quasi-random number generator state | | | |
| | Quasi-random number generator algorithms | | | |

## Random Number Distributions

| | | | | |
|---|---|---|---|---|
| > | The Gaussian Distribution | R* | C | H |
| > | The Gaussian Tail Distribution | R* | C | H |
| > | The Bivariate Gaussian Distribution | R* | | H |
| > | The Exponential Distribution | R* | C | H |
| | The Laplace Distribution | | | |
| | The Exponential Power Distribution | | | |
| | The Cauchy Distribution | | | |
| | The Rayleigh Distribution | | | |
| | The Rayleigh Tail Distribution | | | |
| > | The Landau Distribution | R* | C | H |
| | The Levy alpha-Stable Distributions | | | |
| | The Levy skew alpha-Stable Distribution | | | |
| > | The Gamma Distribution | R* | C | |
| > | The Flat (Uniform) Distribution | R* | C | H |
| > | The Lognormal Distribution | R* | | H |
| > | The Chi-squared Distribution | R* | | H |
| | The F-distribution | | | |
| | The t-distribution | | | |
| | The Beta Distribution | | | |
| | The Logistic Distribution | | | |
| | The Pareto Distribution | | | |
| | The Spherical Distribution (2D & 3D) | | | |
| | The Weibull Distribution | | | |
| | The Type-1 Gumbel Distribution | | | |

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| action | description | Available | | |
|---|---|---|---|---|
| | The Type-2 Gumbel Distribution | | | |
| | The Dirichlet Distribution | | | |
| | General Discrete Distributions | | | |
| > | The Poisson Distribution | R* | C | H |
| | The Bernoulli Distribution | | | |
| > | The Binomial Distribution | R* | | H |
| > | The Multinomial Distribution | R* | | H |
| | The Negative Binomial Distribution | | | |
| | The Pascal Distribution | | | |
| | The Geometric Distribution | | | |
| | The Hypergeometric Distribution | | | |
| + | The Logarithmic Distribution | | | |
| | Shuffling and Sampling | | | |

## Statistics

| | | | | |
|---|---|---|---|---|
| > | Mean, Standard Deviation and Variance | R | | |
| > | Absolute deviation | R | | |
| | Higher moments (skewness and kurtosis) | | | |
| + | Autocorrelation | | | |
| + | Covariance | | | |
| | Weighted Samples | | | |
| > | Maximum and Minimum values | R | | |
| > | Median and Percentiles | R* | | |

## Simulated Annealing

| | | | | |
|---|---|---|---|---|
| | Simulated Annealing algorithm | | | |
| | Simulated Annealing functions | | | |

## Ordinary Differential Equations

| | | | | |
|---|---|---|---|---|
| | Defining the ODE System | | | |
| + | Stepping Functions | | | |
| + | Adaptive Step-size Control | | | |
| | Evolution | | | |

## Interpolation

| | | | | |
|---|---|---|---|---|
| > | Interpolation Functions | | C | |
| | Interpolation Types | | C | |
| | Index Look-up and Acceleration | | | |
| + | Evaluation of Interpolating Functions | | | |
| | Higher-level Interface | | | |

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| action | description | Available | | |
|---|---|---|---|---|
| **Numerical Differentiation** | | | | |
| | Functions | | | |
| **Chebyshev Approximations** | | | | |
| | The gsl_cheb_series struct | | | |
| > | Creation and Calculation of Chebyshev Series | | C | |
| > | Chebyshev Series Evaluation | | C | |
| > | Derivatives and Integrals | | C | |
| **Series Acceleration** | | | | |
| | Acceleration functions | | | |
| | Acceleration functions without error estimation | | | |
| **Discrete Hankel Transforms** | | | | |
| | Definitions | | | |
| | Functions | | | |
| **One dimensional Root-Finding** | | | | |
| > | Initializing the Solver | | | |
| | Providing the function to solve | | | |
| | Search Bounds and Guesses | | | |
| > | Iteration | R* | C | |
| + | Search Stopping Parameters | | | |
| + | Root Bracketing Algorithms | | | |
| > | Root Finding Algorithms using Derivatives | R* | C | |
| **One dimensional Minimization** | | | | |
| | Initializing the Minimizer | R* | C | |
| | Providing the function to minimize | R* | C | |
| | Iteration | R* | C | |
| | Stopping Parameters | R* | C | |
| | Minimization Algorithms | R* | C | |
| **Multidimensional Root-Finding** | | | | |
| | Initializing the Solver | | | |
| | Providing the function to solve | | | |
| + | Iteration | | | |
| + | Search Stopping Parameters | | | |
| + | Algorithms using Derivatives | | | |

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| action | description | Available | | |
|---|---|---|---|---|
| + | Algorithms without Derivatives | | | |
| **Multidimensional Minimization** | | | | |
| | Initializing the Multidimensional Minimizer | R* | C | |
| | Providing a function to minimize | R* | C | |
| | Iteration | R* | C | |
| | Stopping Criteria | R* | C | |
| | Algorithms | R* | C | |
| | Computing the covariance matrix of best fit parameters | R* | C | |
| **Least-Squares Fitting** | | | | |
| > | Linear regression | R* | C | |
| > | Linear fitting without a constant term | R* | C | |
| > | Multi-parameter fitting | R* | C | |
| **Nonlinear Least-Squares Fitting** | | | | |
| | Initializing the Solver | | | |
| | Providing the Function to be Minimized | R* | C | |
| | Iteration | R* | C | |
| | Search Stopping Parameters | R* | C | |
| | Minimization Algorithms using Derivatives | R* | C | |
| | Minimization Algorithms without Derivatives | R* | C | |
| | Computing the covariance matrix of best fit parameters | R* | C | |
| **Physical Constants** | | | | |
| > | Fundamental Constants | R | | |
| | Astronomy and Astrophysics | | | |
| > | Atomic and Nuclear Physics | R | | |
| | Measurement of Time | | | |
| | Imperial Units | | | |
| | Nautical Units | | | |
| | Printers Units | | | |
| | Volume | | | |
| > | Mass and Weight | R | | |
| | Thermal Energy and Power | | | |
| > | Pressure | R | | |
| | Viscosity | | | |
| | Light and Illumination | | | |
| | Radioactivity | | | |
| | Force and Energy | | | |

# 4 clhep

The CLHEP library has three mayor areas of interest:

VECTOR (more functionality in TVector2, TVector3)

MATRIX (poor package compared to ROOT TMatrix package)

RANDOM (Most classes in TRandom/TRandom3). Missing functions trivial to implement via TF1. marked with "+" could be taken)

| Vector | | | | |
|---|---|---|---|---|
| action | description | Available | | |
| | Hep2Vector | R* | | |
| | Hep3RotationInterface | R* | | |
| | Hep3Vector | R* | | |
| | Hep4RotationInterface | R* | | |
| | HepAxisAngle | R* | | |
| | HepBoost | R* | | |
| | HepBoostX | R* | | |
| | HepBoostY | R* | | |
| | HepBoostZ | R* | | |
| | HepEulerAngles | R* | | |
| | HepLorentzRotation | R* | | |
| | HepLorentzVector | R* | | |
| | HepRotation | R* | | |
| | HepRotationX | R* | | |
| | HepRotationY | R* | | |
| | HepRotationZ | R* | | |
| Matrix | | | | |
| | HepDiagMatrix | R* | | |
| | HepGenMatrix | R* | | |
| | HepMatrix | R* | | |
| | HepPile | R* | | |
| | HepSymMatrix | R* | | |
| | HepVector | R* | | |
| Random | | | | |

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| action | description | Available | | |
|---|---|---|---|---|
| | DRand48Engine | | | |
| | DualRand | | | |
| | HepJamesRandom | | | |
| | HepRandom | | | |
| | HepRandomEngine | R* | | |
| | HepStat | | | |
| | Hurd160Engine | | | |
| | Hurd288Engine | | | |
| | MTwistEngine | R* | | |
| | NonRandomEngine | | | |
| | RandBinomial | R* | | |
| + | RandBit | | | |
| + | RandBreitWigner | | | |
| + | RandChiSquare | | | |
| | RandEngine | | | |
| | RandExponential | R* | | |
| | RandFlat | R* | | |
| + | RandGamma | | | |
| | RandGauss | R* | | |
| + | RandGaussQ | | | |
| + | RandGaussT | | | |
| | RandGeneral | | | |
| | RandLandau | R* | | |
| | RandPoisson | R* | | |
| | RandPoissonQ | | | |
| | RandPoissonT | | | |
| + | RandStudentT | | | |
| + | RanecuEngine | | | |
| + | Ranlux64Engine | | | |
| | RanluxEngine | | | |
| + | RanshiEngine | | | |
| | TripleRand | | | |
| | HepRandomVector | | | |
| | RandMultiGauss | | | |

# 5 root

R only static functions, not depending on the framework

R* A set of classes using the framework (I/O, visualisation) The following classes can be used via CINT, they are persistent-capable. They hidde low-level math functions in high level objects.

## Basic Math (R)

| class | functionality |
|---|---|
| TMath | only static functions, not depending on the framework<br>Fundamental constants<br>Trigo (Sin, Cos, CosH, ..)<br>Misc (Factorial, Nint, IsFinite, IsNaN, etc)<br>Abs, Even, Odd, NextPrime, Sign<br>Min, Max, LocMin, LocMax<br>Range, BinarySearch<br>Hash, IsInside<br>CrossProduct, Normal2Plane, etc<br>Erf, Erfc,Freq, Prob,<br>Gaus, Gamma, BreitWigner,Landau,LnGamma,Poisson<br>KolmogorovProb, Voigt<br>Bessel functions I0,K0,I1,K1,J0,J1,Y0,Y1<br>Struve functions H0,H1,L0,L1 |

## General Functions (R*)

| | |
|---|---|
| TFormula | parametric expression evaluator (1,2,3,4-dim) |
| TF1 | 1-dim parametric function<br>Any analytic & parametric expressions<br>Any interpreted function<br>Any compiled function with a dictionary<br>Derivatives<br>Integrals<br>Zero Finding<br>Min/Max Finding<br>Random number generation<br>The main interface for histogram/graph fitting<br>Visualisation |
| TF2 | |

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| class | functionality |
|---|---|
| | same as TF1 for 2-d parametric functions |
| TF12 | Projection of a TF2 along x or y |
| TF3 | same as TF1 for 3-d parametric functions |

## Random Numbers (R*)

| | |
|---|---|
| TRandom | Flat, Uniform, Binomial, Exp, Landau, Poisson |
| TRandom3 | same with Mersenne Twister |
| Random numbers from TF1, TF2 and TF3<br>Random numbers from histograms TH1, TH2, TH3 or TProfile | |

## Sets of Points (R*)

| | |
|---|---|
| TGraph | Drawing, smoothing, fitting, interpolation of n,x[i],y[i] |
| TGraph2D | same for n,x[i],y[i],z[i]<br>Delaunay triangulation & interpolation |
| TSpline3 | Cubic spline interpolation, smoothing, visualisation |
| TSpline5 | same with quintic splines |

## Linear Algebra (R*)

| | |
|---|---|
| TMatrix ++ | Performant linear algebra package<br>supporting many matrix types<br>Decomposition, SVD<br>Eigenvalues, etc |
| TVector | All operations on vectors or vectors with matrices |
| TCL | A translation in C++ of frequently used routines from Cernlib packages F110, F112 |

## Physics Vectors (R*)

| | |
|---|---|
| TLorentzVector | |

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| | |
|---|---|
| | continued from previous page |
| class | functionality |
| TLorentzRotation | |
| TRotation | |
| TVector2 | |
| TVector3 | |
| Fitting, Clusters (R*) | |
| TFumili | Fitting histograms and graphs with Fumili |
| TMinuit | same with Minuit |
| TMultiDimFit | Multi-dim parametrisation and fitting |
| TMultiLayerPerceptron | Neural network (clustering & fit) |
| TPrincipal | Principal component analysis |

## References


[1] CERNLIB *http://cernlib.web.cern.ch/cernlib/kernlib.html*

[2] GNU *Scientific Library*, http://www.gnu.org/software/gsl/

[3] CLHEP *A Class Library for High-Energy Physics*, http://wwwasd.web.cern.ch/wwwasd/lhc++/clhep/ .

[4] ROOT *An Object-Oriented Data Analysis Framework*, *http://root.cern.ch/*

[5] Lorenzo Moneta, *Feedback from LHC experiments*, *http://proj-clhep.web.cern.ch/proj-clhep/Workshop-2003/*